

## 1

## Rational and Irrational numbers



Let's recall.

We are familiar with Natural numbers, Whole numbers, Integers and Rational numbers.

**Natural numbers**

1, 2, 3, 4, ...

**Whole numbers**

0, 1, 2, 3, 4, ...

**Integers**

..., -4, -3, -2, -1, 0, 1, 2, 3, ...

**Rational numbers** $-\frac{25}{3}, \frac{10}{-7}, -4, 0, 3, 8, \frac{32}{3}, \frac{67}{5}, \text{etc.}$ 

**Rational numbers :** The numbers of the form  $\frac{m}{n}$  are called rational numbers.

Here,  $m$  and  $n$  are integers but  $n$  is not zero.

We have also seen that there are infinite rational numbers between any two rational numbers.

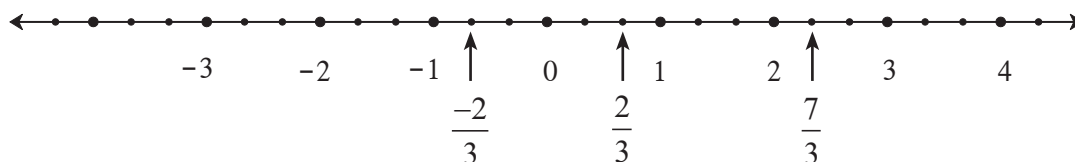


Let's learn.

**To show rational numbers on a number line**

Let us see how to show  $\frac{7}{3}, 2, \frac{-2}{3}$  on a number line.

Let us draw a number line.



- We can show the number 2 on a number line.
- $\frac{7}{3} = 7 \times \frac{1}{3}$ , therefore each unit on the right side of zero is to be divided in three equal parts. The seventh point from zero shows  $\frac{7}{3}$ ; or  $\frac{7}{3} = 2 + \frac{1}{3}$ , hence the point at  $\frac{1}{3}$  rd distance of unit after 2 shows  $\frac{7}{3}$ .



- To show  $\frac{-2}{3}$  on the number line, first we show  $\frac{2}{3}$  on it. The number to the left of 0 at the same distance will show the number  $\frac{-2}{3}$ .

### Practice set 1.1

- Show the following numbers on a number line. Draw a separate number line for each example.

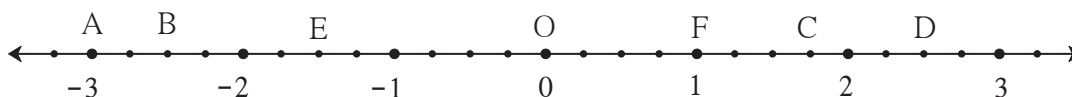
(1)  $\frac{3}{2}, \frac{5}{2}, -\frac{3}{2}$

(2)  $\frac{7}{5}, \frac{-2}{5}, \frac{-4}{5}$

(3)  $\frac{-5}{8}, \frac{11}{8}$

(4)  $\frac{13}{10}, \frac{-17}{10}$

- Observe the number line and answer the questions.



- Which number is indicated by point B?
- Which point indicates the number  $1\frac{3}{4}$ ?
- State whether the statement, 'the point D denotes the number  $\frac{5}{2}$ ', is true or false.



Let's learn.

### Comparison of rational numbers

We know that, for any pair of numbers on a number line the number to the left is smaller than the other. Also, if the numerator and the denominator of a rational number is multiplied by any non zero number then the value of rational number does not change. It remains the same. That is,  $\frac{a}{b} = \frac{ka}{kb}$ , ( $k \neq 0$ ).

**Ex. (1)** Compare the numbers  $\frac{5}{4}$  and  $\frac{2}{3}$ . Write using the proper symbol of  $<$ ,  $=$ ,  $>$ .

**Solution :**  $\frac{5}{4} = \frac{5 \times 3}{4 \times 3} = \frac{15}{12}$        $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

$\frac{15}{12} > \frac{8}{12}$        $\therefore \frac{5}{4} > \frac{2}{3}$



**Ex. (2)** Compare the rational numbers  $\frac{-7}{9}$  and  $\frac{4}{5}$ .

**Solution :** A negative number is always less than a positive number.

$$\text{Therefore, } -\frac{7}{9} < \frac{4}{5}.$$

**To compare two negative numbers,**

let us verify that if  $a$  and  $b$  are positive numbers such that  $a < b$ , then  $-a > -b$ .

$$\left. \begin{array}{l} 2 < 3 \text{ but } -2 > -3 \\ \frac{5}{4} < \frac{7}{4} \text{ but } \frac{-5}{4} > \frac{-7}{4} \end{array} \right\} \text{Verify the comparisons using a number line.}$$

**Ex. (3)** Compare the numbers  $\frac{-7}{3}$  and  $\frac{-5}{2}$ .

**Solution :** Let us first compare  $\frac{7}{3}$  and  $\frac{5}{2}$ .

$$\frac{7}{3} = \frac{7 \times 2}{3 \times 2} = \frac{14}{6}, \quad \frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6} \quad \text{and} \quad \frac{14}{6} < \frac{15}{6}$$
$$\therefore \frac{7}{3} < \frac{5}{2} \quad \therefore \frac{-7}{3} > \frac{-5}{2}$$

**Ex. (4)**  $\frac{3}{5}$  and  $\frac{6}{10}$  are rational numbers. Compare them.

**Solution :**  $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} \quad \therefore \frac{3}{5} = \frac{6}{10}$

**The following rules are useful to compare two rational numbers.**

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are rational numbers such that  $b$  and  $d$  are positive, and

(1) if  $a \times d < b \times c$  then  $\frac{a}{b} < \frac{c}{d}$

(2) if  $a \times d = b \times c$  then  $\frac{a}{b} = \frac{c}{d}$

(3) if  $a \times d > b \times c$  then  $\frac{a}{b} > \frac{c}{d}$

### Practice Set 1.2

**1.** Compare the following numbers.

- |                                      |                                   |                                   |                                  |                                     |
|--------------------------------------|-----------------------------------|-----------------------------------|----------------------------------|-------------------------------------|
| (1) $-7, -2$                         | (2) $0, \frac{-9}{5}$             | (3) $\frac{8}{7}, 0$              | (4) $\frac{-5}{4}, \frac{1}{4}$  | (5) $\frac{40}{29}, \frac{141}{29}$ |
| (6) $-\frac{17}{20}, \frac{-13}{20}$ | (7) $\frac{15}{12}, \frac{7}{16}$ | (8) $\frac{-25}{8}, \frac{-9}{4}$ | (9) $\frac{12}{15}, \frac{3}{5}$ | (10) $\frac{-7}{11}, \frac{-3}{4}$  |





Let's learn.

### Decimal representation of rational numbers

If we use decimal fractions while dividing the numerator of a rational number by its denominator, we get the decimal representation of a rational number. For example,  $\frac{7}{4} = 1.75$ . In this case, after dividing 7 by 4, the remainder is zero. Hence the process of division ends.

Such a decimal form of a rational number is called a terminating decimal form.

We know that every rational number can be written in a non-terminating recurring decimal form.

**For example,** (1)  $\frac{7}{6} = 1.1666... = 1.1\dot{6}$

(2)  $\frac{5}{6} = 0.8333... = 0.8\dot{3}$

(3)  $\frac{-5}{3} = -1.666... = -1.\dot{6}$

(4)  $\frac{22}{7} = 3.142857142857... = 3.\overline{142857}$  (5)  $\frac{23}{99} = 0.2323... = 0.\overline{23}$

Similarly, a terminating decimal form can be written as a non-terminating recurring decimal form. For example,  $\frac{7}{4} = 1.75 = 1.75000... = 1.75\dot{0}$ .

### Practice Set 1.3

1. Write the following rational numbers in decimal form.

(1)  $\frac{9}{37}$

(2)  $\frac{18}{42}$

(3)  $\frac{9}{14}$

(4)  $\frac{-103}{5}$

(5)  $-\frac{11}{13}$



Let's learn.

### Irrational numbers

In addition to rational numbers, there are many more numbers on a number line. They are not rational numbers, that is, they are irrational numbers.  $\sqrt{2}$  is such an irrational number.

We learn how to show the number  $\sqrt{2}$  on a number line.

- On a number line, the point A shows the number 1. Draw line  $l$  perpendicular to the number line through point A.

Take point P on line  $l$  such that  $OA = AP = 1$  unit.

- Draw seg OP. The  $\triangle OAP$  formed is a right angled triangle.



By Pythagoras theorem,

$$OP^2 = OA^2 + AP^2$$

$$= 1^2 + 1^2 = 1 + 1 = 2$$

$$OP^2 = 2$$

$$\therefore OP = \sqrt{2} \text{ ... (taking square roots on both sides)}$$

- Now, draw an arc with centre O and radius OP. Name the point as Q where the arc intersects the number line. Obviously distance OQ is  $\sqrt{2}$ . That is, the number shown by the point Q is  $\sqrt{2}$ .

If we mark point R on the number line to the left of O, at the same distance as OQ, then it will indicate the number  $-\sqrt{2}$ .

We will prove that  $\sqrt{2}$  is an irrational number in the next standard. We will also see that the decimal form of an irrational number is non-terminating and non-recurring.

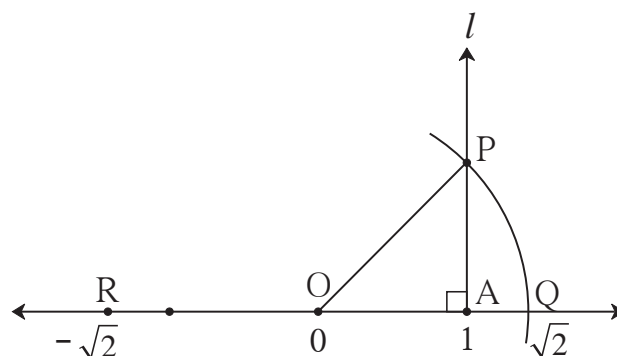
### Note that -

In the previous standard we have learnt that  $\pi$  is not a rational number. It means it is irrational. For calculation purpose we take its value as  $\frac{22}{7}$  or 3.14 which are very close to  $\pi$ ; but  $\frac{22}{7}$  and 3.14 are rational numbers.

The numbers which can be shown by points of a number line are called real numbers. We have seen that all rational numbers can be shown by points of a number line. Therefore, all rational numbers are real numbers. There are infinitely many irrational numbers on the number line.

$\sqrt{2}$  is an irrational number. Note that the numbers like  $3\sqrt{2}$ ,  $7 + \sqrt{2}$ ,  $3 - \sqrt{2}$  etc. are also irrational numbers; because if  $3\sqrt{2}$  is rational then  $\frac{3\sqrt{2}}{3}$  should also be a rational number, which is not true.

We learnt to show rational numbers on a number line. We have shown the irrational number  $\sqrt{2}$  on a number line. Similarly we can show irrational numbers like  $\sqrt{3}$ ,  $\sqrt{5}$  ... on a number line.

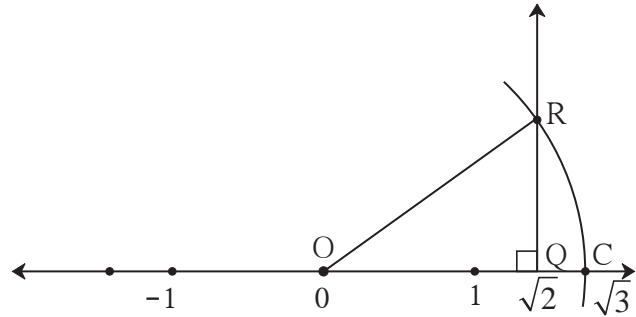


### Practice Set 1.4

- The number  $\sqrt{2}$  is shown on a number line. Steps are given to show  $\sqrt{3}$  on the number line using  $\sqrt{2}$ . Fill in the boxes properly and complete the activity.

### Activity :

- The point Q on the number line shows the number .....
- A line perpendicular to the number line is drawn through the point Q. Point R is at unit distance from Q on the line.
- Right angled  $\Delta$  ORQ is obtained by drawing seg OR.
- $l(OQ) = \sqrt{2}$  ,  $l(QR) = 1$



$\therefore$  by Pythagoras theorem,

$$\begin{aligned}
 [l(OR)]^2 &= [l(OQ)]^2 + [l(QR)]^2 \\
 &= \boxed{\phantom{00}}^2 + \boxed{\phantom{00}}^2 = \boxed{\phantom{00}} + \boxed{\phantom{00}} \\
 &= \boxed{\phantom{00}} \quad \therefore l(OR) = \boxed{\phantom{00}}
 \end{aligned}$$

Draw an arc with centre O and radius OR. Mark the point of intersection of the line and the arc as C. The point C shows the number  $\sqrt{3}$ .

2. Show the number  $\sqrt{5}$  on the number line.
- 3★. Show the number  $\sqrt{7}$  on the number line.



### Answers

#### Practice Set 1.1

2. (1)  $\frac{-10}{4}$  (2) C (3) True

#### Practice Set 1.2

1. (1)  $-7 < -2$  (2)  $0 > \frac{-9}{5}$  (3)  $\frac{8}{7} > 0$  (4)  $\frac{-5}{4} < \frac{1}{4}$  (5)  $\frac{40}{29} < \frac{141}{29}$
- (6)  $\frac{-17}{20} < \frac{-13}{20}$  (7)  $\frac{15}{12} > \frac{7}{16}$  (8)  $\frac{-25}{8} < \frac{-9}{4}$  (9)  $\frac{12}{15} > \frac{3}{5}$
- (10)  $\frac{-7}{11} > \frac{-3}{4}$

#### Practice Set 1.3

- (1)  $0.\overline{243}$  (2)  $0.\overline{428571}$  (3)  $0.\overline{6428571}$  (4)  $-20.\overline{6}$  (5)  $-0.\overline{846153}$

